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ON ANALYSIS OF THE FRÉCHET DERIVATIVES

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Dedicated to Prof. A.K. Agarwal on his 70^{th} ***Birth Anniversary***

Abstract: This work reviews various aspects of the Fréchet derivatives basically from the first principle through some numerous results bordering on the basic theories. On displaying thorough proofs to the numerous analytical results, we hence showed that the Fréchet derivatives is not a number at all but a linear operator and in the end we succeeded in establishing the existence of various generalized results on the Fréchet derivatives.

Keyword and Phrases: Banach Space, Operator, Derivatives, Chain Rule, Mean Value Theorem and Implicit Function Theorem.

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1. Introduction

Definition 1.1. *Let T be an operator mapping a Banach space X into a Banach space Y. If there is a bounded linear operator φ from S a bounded subset of X into Y such that*

$$T\left(x+\Delta x\right)-T\left(x\right)=T'\left(x\right),\Delta x+\varphi\left(x,\Delta x\right),$$

, and

$$\underset{\|x\|\to 0}{Lim}\frac{\|\varphi\left(x,\Delta x\right)\|}{\|\Delta x\|}=0 \tag{1.1A}$$

or

$$\underset{\|\Delta x\|\to 0}{Lim}\frac{\|T\left(x_0+\Delta x\right)-T\left(x_0\right)-\varphi\left(\Delta x\right)\|}{\|\Delta x\|}=0 \tag{1.1B}$$